## THE LEBANESE AMERICAN UNIVERSITY Fall 2014 MTH305 EXAM 2

November 26, 2014

Name:

ID:

This exam is comprised of 10 problems. Answer the questions in the space provided for each problem. If more space is needed, use the back of the page. Make sure to justify all your answers.

## **Duration: 60 minutes**

1. (10 pts.) The probability distribution of a discrete random variable X is given by:

| _          | Х    | -2k     | 6 | k   | 15  |
|------------|------|---------|---|-----|-----|
| _          | f(x) | s - 0.1 | S | 0.3 | 0.3 |
| where k>0. |      |         |   |     |     |

**a.** Determine the expected value of X. Hint: first you have to find s.

**b.** If  $\sigma_X^2 = 63$ , determine the value of k

2. (10 pts.) The probability density function of a continuous random variable X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1\\ 0 & Otherwise \end{cases}$$

a. Calculate the Expected Value of  $\operatorname{RV} X$  .

b. Calculate  $\sigma_Y^2$  where Y = 3X - 1

- 3. (10 pts) Seventy new jobs are opening up at an automobile manufacturing plant, and 1500 applicants show up for the 70 positions. To select the best 70 from among the applicants, the company gives a test that covers mechanical skill, manual dexterity, and mathematical ability. The mean grade on this test turns out to be 64 and the scores have a standard deviation of 6.
  - a. Can you tell, using Chebyshev's theorem, whether a person who scores 84 can be selected to one of the jobs? Justify your answer

b. Solve part (a) assuming that the distribution is symmetric about the mean. Justify your answer

- 4. (10 pts.)
  - a. If a, b, c and d are constants, show, starting from the definition of covariance, that:  $Cov(aX + b, cY + d) = ac\sigma_{XY}$

b. If  $\sigma_{XY} = 12$ , then Cov(-2X + 14,7Y - 151) =\_\_\_\_\_.

5. (10 pts.) The probabilities are 0.4, 0.2, 0.3, and 0.1, respectively, that an MTH305 student gets D, C, B or A on this course. What is the probability that among 20 randomly selected MTH305 students, 3 get A, 5 get B, 4 get C and 8 get D?

- 6. (10 pts.) Suppose the probability that any given person will believe a rumor about a famous actress is 0.8.
  - a. What is the probability that the sixth person to hear this rumor is the fourth one to believe it?

b. What is the probability that from a random sample of 8 persons who heard this rumor, at most 7 will believe it?

7. (10 pts) The number of car accidents at a certain intersection follows a poisson distribution with a mean of  $\lambda$  accidents per month. ( $\lambda > 0$ ) If the probability of having 3 accidents per month **is double** the probability of having 4 accidents per month, find the probability of having 7 accidents per month.

- 8. (10 pts) The number of clients visiting Blanca pharmacy follows the Poisson distribution with a mean of 168 clients per WEEK.
  - a. Calculate the probability that in a given day 31 clients visit Blanca Pharmacy

b. Calculate the probability that in 3 out of 8 consecutive days, exactly 31 clients (per day) visit the pharmacy.

9. (10 pts.) If a, b are constants, show, starting from the definition of variance, that:  $\sigma^2_{aX+b} = a^2 \sigma_X^2$ 

10. (10 pts.)

a. Find the parameters, n and p, of a binomial random Variable with mean 80 and standard deviation 4.

b. You are flipping an unbalanced coin with P(T)=0.1666666. On which flip do you expect to get the first tail?